CALCULATION OF A CRACKED ANISOTROPIC PLATE STRENGTHENED WITH FASTENED AND BONDED RIBS

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The wide application of composite materials in various branches of technology promoted the rapid development of bonding technology, the increased complexity of structures with bonded and bonded-mechanical joints, and the refinement of calculational models to evaluate the stress–strain state and the residual strength of the structures. A significant amount of research, mainly with finite elements, has been devoted to analyzing the stress–strain state of isotropic or anisotropic platelike structural elements with defects (cracks, ruptures) and bonded reinforcing elements (crack stiffeners, repair straps) (see [1-3] for an example and the bibliographies therein). Nonetheless, there are practically no publications on similar problems for bonded-mechanical (riveted and bonded or bolted and bonded) joints.

Here we construct a general system of integral equations for an elastic rectilinear anisotropic semi-infinite plate weakened by a system of curvilinear cuts and strengthened by bonded or riveted and bonded ribs. An algorithm is suggested for a numerical solution. Several calculated results are presented and compared with experimental data.

1. We examine a semi-infinite elastic plate of constant thickness, made from a rectilinear anisotropic material, which has one plane of elastic symmetry parallel to the midplane of the plate $D = \{-\infty < y < \infty,\ x > 0\}$. The plate is weakened by a system of arbitrarily oriented smooth curvilinear cuts all the way through $L_j = \{t = t^j(\eta) \,|\, |\eta| < 1\}$ $(j = \overline{1, k})$. The cuts Γ_s $(s = \overline{1, m})$ form angles ϑ_s with the x axis (Fig. 1a). Strengthening elements (stiffening ribs) (Fig. 1b) are fastened along the cuts with a bond layer with shear modulus G_s and thickness Δ_s and also with pins of diameter d (Fig. 1b). The pins are numbered from 1 to N^s on each rib. We denote the pliance of the pins by q^s, and the coordinates of the pin centers by t_i^s $(i = \overline{1, N^s})$. We limit ourselves to the case where an external load field, given by stresses at infinity, acts on the elastic system in the plate. Concentrated forces $P_s^1 \exp[i(\vartheta_s + \pi)]$ and $P_s^2 \exp[i\vartheta_s]$ are applied to the ends $t^s(-1)$ and $t^s(1)$ of the ribs. The edges of the cuts are free from loads. Here n is the right-hand normal to the circulation of the lines $L = \bigcup_{j=1}^{k} L_j$ and $\Gamma = \bigcup_{s=1}^{m} \Gamma_s$.

We make a series of simplifying assumptions [4]: The thickness of the plate and the dimensions of the cross section of the bond layer and the ribs are small compared to the line $2\rho_s$ of the joint section Γ_s. The plate is in overall state of plane stress. Weakening of the plate and ribs due to the placement of the pins is not considered.

If a crack passes through a pin hole, we do not consider the effect of these holes and the pins that fill them. The resultant errors are insignificant if the length of the crack, which goes from the pin hole, is larger than the hole radius. The pin deforms elastically only along its axis: the shear rigidity of the rib is negligibly small. We assume that the rib is joined smoothly to the plate $L \cap \Gamma = 0$ (a generalization to the case where the crack extends under a rib is given below). The bond works in shear; the strains $\gamma_s(t)$ and stresses $\tau^s(t)$ are functions of the longitudinal coordinate. Detachment stresses arising in the bond are neglected.

By a pinned attachment we mean any industrial operation or method for point fastening (welding, riveting, bolting), where the dimensions of the coupling area are small compared to both the characteristic dimensions of the body and the step c_s between attachments. All attachments are identical. The action of the pin on the plate is modeled by the load of its constant tangential forces τ_i^s $(i = \overline{1, N^s},\ s = \overline{1, m})$ over a square area S_i (see Fig. 1b) with side d.

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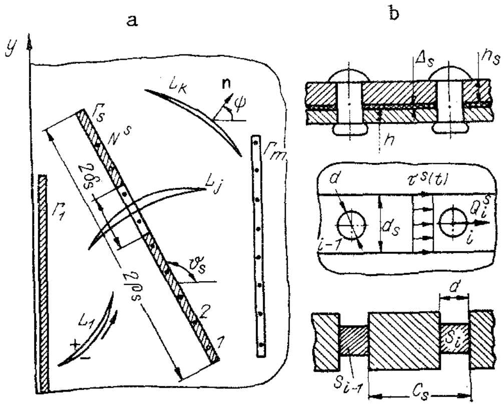

Fig. 1

The total force vector on each pin

$$Q_i^s = \int\limits_{S_i} \tau_i^s \, ds = \tau_i^s d^2 \quad \left(i = \overline{1, N^s},\ s = \overline{1, m}\right) \tag{1.1}$$

and the contact forces $r(t) = \{r_s(t) = \tau^s(t) d_s |\ t \in \Gamma_s,\ s = \overline{1, m}\}$ transmitted from the ribs to the plate along the current Γ_s will be considered as volume forces in the plate which coincide with its midplane (eccentricity effects are not considered). The positive direction for Q_i^s and $r_s(t)$ coincides with the direction of the vector $\exp[i(\vartheta_s + \pi)]$ $(s = \overline{1, m})$.

The stresses and displacements in the anisotropic plate are expressed through two analytical functions $\Phi_\nu(z_\nu)$ ($\nu = 1, 2$) [5]

$$(\sigma_x, \sigma_y, \tau_{xy}) = 2\mathrm{Re}\left\{\sum_{\nu=1}^{2} \left(\mu_\nu^2, 1, -\mu_\nu\right) \Phi_\nu(z_\nu)\right\},\ (u, v) = 2\ \mathrm{Re}\left\{\sum_{\nu=1}^{2} (p_\nu, q_\nu)\, \varphi_\nu(z_\nu)\right\}, \tag{1.2}$$

which satisfy a given system of external loads applied to the plate, the boundary conditions

$$\sigma_x^- |_{x=0} = \tau_{xy}^- |_{x=0} = 0,\ \tau_n^- - \tau_n^+ = r(t)/h,\ \sigma_n^- = \sigma_n^+,\ u^- = u^+,\ v^- = v^+,\ t \in \Gamma; \tag{1.3}$$

$$a(t)\,\Phi_1^\pm(t_1) + b(t)\,\overline{\Phi_1^\pm(t_1)} + \Phi_2^\pm(t_2) = 0,\ t \in L, \tag{1.4}$$

$$a(t) = \frac{\mu_1 - \bar{\mu}_2}{\mu_2 - \bar{\mu}_2} \frac{M_1(t)}{M_2(t)},\ b(t) = \frac{\bar{\mu}_1 - \bar{\mu}_2}{\mu_2 - \bar{\mu}_2} \frac{\overline{M_1(t)}}{M_2(t)},\ M_\nu(t) = \mu_\nu \cos\psi - \sin\psi$$

and the compatibility conditions between the displacements of the rib and the plate along the contact curve Γ_s in the bond layer [1, 4]

$$w(t^s, \vartheta_s) - w_s(t^s) = \frac{\Delta_s}{G_s d_s} r_s(t^s) \quad (t^s \in \Gamma_s',\ s = \overline{1, m}) \tag{1.5}$$

and at the pinning points t_i^s [4]

$$w(t_i^s, \vartheta_s) - w_s(t_i^s) = q^s Q_i^s \quad \left(t_i^s \in \Gamma_s'',\ i = \overline{1, N^s},\ s = \overline{1, m}\right),$$

$$\Gamma_s'' = \bigcup_{i=1}^{N^s} \{|t_i^s - t| \leqslant d/2\},\ \Gamma_s' = \Gamma_s \diagdown \Gamma_s''. \tag{1.6}$$

Here $\varphi_\nu(z_\nu)$ is the inverse image of $\Phi_\nu(z_\nu)$; $\sigma_n^\pm$, $\tau_n^\pm$, $u^\pm$, and $v^\pm$ are the left and right limiting values of the stresses and displacements on Γ; $\psi = \psi(t)$ is the angle between the normal n at the point $t \in L$ and the x axis; $w(t, \vartheta_s)$ is the displacement of the plate at point t in the direction $\exp(i\vartheta_s)$; $w_s(t)$ is the displacement of the s-th rib.

2. We use the superposition principle to find the function $\Phi_\nu(z_\nu)$ which solves the boundary problem (1.2)-(1.6) in the form

$$\Phi_\nu(z_\nu) = \sum_{j=1}^{3} \Phi_\nu^j(z_\nu), \tag{2.1}$$

where $\Phi_\nu^1(z_\nu)$ is the solution for the unstrengthened plate without cracks which satisfies the boundary conditions on the edge of the half-plane and at infinity; it can be determined by standard methods [5].

Following [4, 6], we represent $\Phi_\nu^j(z_\nu)$ ($j = 2, 3$) in the form of a generalized Cauchy integral, whose integrand is the fundamental solution to the action of a single concentrated force $\exp(i\varphi)$ at the point τ of an anisotropic plate [5]:

$$\Phi_\nu^2(z_\nu) = \frac{1}{2\pi i}\int_\Gamma \left\{\frac{A_\nu(\tau)}{z_\nu - \tau_\nu} + \frac{l_\nu s_\nu \overline{A_1(\tau)}}{s_\nu z_\nu - \bar\tau_1} + \frac{n_\nu m_\nu \overline{A_2(\tau)}}{m_\nu z_\nu - \bar\tau_2}\right\} r(\tau)\, ds,$$
$$\Phi_\nu^3(z_\nu) = \frac{1}{2\pi i}\int_L \left\{\frac{\omega_\nu(\tau)\, d\tau_\nu}{\tau_\nu - z_\nu} - \frac{l_\nu s_\nu \overline{\omega_1(\tau)}\, d\bar\tau_1}{\bar\tau_1 - s_\nu z_\nu} - \frac{n_\nu m_\nu \overline{\omega_2(\tau)}\, d\bar\tau_2}{\bar\tau_2 - m_\nu z_\nu}\right\}. \quad (2.2)$$

The complex function $\omega_\nu(t) = \{\omega_{\nu j}(t) \mid t \in L_j,\ j = \overline{1,\ k}\}$ and the contact forces $r(t) = \{r_s(t) \mid t \in \Gamma_s,$ $s = \overline{1,\ m}\}$ are the main unknowns of the problem. The constants $A_\nu(t)$ $(t \in \Gamma_s)$, l_ν, s_ν, n_ν, m_ν are defined in [6].

The functions $\Phi_\nu^j(z_\nu)$ ($j = 2, 3$), which are expressed in the form of (2.2), automatically satisfy the boundary conditions (1.3) and the given forces at infinity.

From (1.2), (2.1), and (2.2), the displacements of the plate and the ribs can be written as

$$w(t, \vartheta_s) = w^0(t, \vartheta_s) - \mathrm{Re}\left\{\sum_{\nu=1}^{2} T_\nu(\vartheta_s)\left[\frac{1}{\pi i}\int_L [\omega_\nu(\tau)\ln(\tau_\nu - t_\nu)\, d\tau_\nu - \right.\right.$$
$$\left. - l_\nu \ln(\bar\tau_1 - s_\nu t_\nu)\overline{\omega_1(\tau)}\, d\bar\tau_1 - n_\nu \ln(\bar\tau_2 - m_\nu t_\nu)\overline{\omega_2(\tau)}\, d\bar\tau_2\right] -$$
$$\left. - \frac{1}{\pi i}\int_\Gamma [A_\nu(\tau)\ln(\tau_\nu - t_\nu) + l_\nu\overline{A_1(\tau)}\ln(\bar\tau_1 - s_\nu t_\nu) + n_\nu\overline{A_2(\tau)}\ln(\bar\tau_2 - m_\nu t_\nu)] r(\tau)\, ds]\right\},$$
$$w_s(t) = \frac{P_s^1}{F_s E_s}(t + \rho_s) + \int_{-\rho_s}^{t} \frac{dv}{E_s F_s}\int_v^{\rho_s} r_s(u)\, du + C_s \quad (s = \overline{1,\ m}), \quad (2.3)$$

where C_s is the displacement of the rib as a rigid body; E_s and F_s are Young's modulus and the cross-sectinal area, respectively, of the s-th rib; and $w^0(t, \vartheta_s)$ is the displacement from the action of all the external forces. The values of $w^0(t, \vartheta_s)$ will be considered unknown.

By substituting (2.1) into the boundary conditions (1.4), we obtain

$$\int_L \frac{\omega_1(\tau)\, d\tau_1}{\tau_1 - t_1} + \int_L \{k_1(t, \tau)\omega_1(\tau) + k_2(t, \tau)\overline{\omega_1(\tau)}\}\, ds + \int_\Gamma k_3(t, \tau) r(\tau)\, ds = f_1(t),$$
$$t \in L,$$
$$\omega_2(t) = -a(t)\omega_1(t) - b(t)\overline{\omega_1(t)},$$
$$k_1(t, \tau) = \frac{1}{2}\left\{\frac{d}{ds}\left[\ln\frac{\bar\tau_2 - \bar t_2}{\tau_1 - t_1}\right] + \frac{\overline{b(\tau)} - \overline{b(t)}}{\overline{b(t)}(\bar\tau_2 - \bar t_2)}\frac{d\bar\tau_2}{ds} + \frac{\overline{a(t)}}{\overline{b(t)}}\frac{\bar l_1 \bar s_1}{\tau_1 - \bar s_1 \bar t_1}\frac{d\tau_1}{ds} - \right.$$
$$\left. - \frac{\overline{a(t)}\, a(\tau)}{\overline{b(t)}}\frac{\bar n_1 \bar m_1}{\tau_2 - \bar m_1 \bar t_1}\frac{d\tau_2}{ds} + \frac{b(\tau)\, n_1 m_1}{\bar\tau_2 - m_1 t_1}\frac{d\bar\tau_2}{ds} + \frac{1}{\overline{b(t)}}\frac{\bar l_2 \bar s_2}{\tau_1 - \bar s_2 \bar t_2}\frac{d\tau_1}{ds} - \frac{a(\tau)}{\overline{b(t)}}\frac{\bar n_2 \bar m_2}{\tau_2 - \bar m_2 \bar t_2}\frac{d\tau_2}{ds}\right\},$$
$$k_2(t, \tau) = \frac{1}{2}\left\{\frac{\overline{a(t)}}{\overline{b(t)}}\frac{d}{ds}\left[\ln\frac{\bar\tau_2 - \bar t_2}{\bar\tau_1 - \bar t_1}\right] + \frac{\overline{a(\tau)} - \overline{a(t)}}{\overline{b(t)}(\bar\tau_2 - \bar t_2)}\frac{d\bar\tau_2}{ds} - \frac{\overline{a(t)}\, b(\tau)}{\overline{b(t)}}\frac{n_1 m_1}{\bar\tau_2 - m_1 \bar t_1}\frac{d\bar\tau_2}{ds} - \right.$$
$$\left. - \frac{l_1 s_1}{\bar\tau_1 - s_1 t_1}\frac{d\bar\tau_1}{ds} + \frac{\overline{a(\tau)}\, n_1 m_1}{\bar\tau_2 - m_1 t_1}\frac{d\bar\tau_2}{ds} - \frac{b(\tau)}{\overline{b(t)}}\frac{\bar n_2 \bar m_2}{\tau_2 - \bar m_2 \bar t_2}\frac{d\tau_2}{ds}\right\}, \quad (2.4)$$
$$k_3(t, \tau) = R_1(t, \tau) - \frac{\overline{a(t)}}{\overline{b(t)}}\overline{R_1(t, \tau)} - \frac{1}{\overline{b(t)}}\overline{R_2(t, \tau)},$$
$$f_1(t) = -\pi i\left[\frac{\overline{a(t)}}{\overline{b(t)}}\overline{\Phi_1^0(t_1)} + \Phi_1^0(t_1) + \frac{1}{\overline{b(t)}}\overline{\Phi_2^0(t_2)}\right],$$
$$R_\nu(t, \tau) = \frac{A_\nu(\tau)}{t_\nu - \tau_\nu} + \frac{l_\nu s_\nu \overline{A_1(\tau)}}{s_\nu t_\nu - \bar\tau_1} + \frac{n_\nu m_\nu \overline{A_2(\tau)}}{m_\nu t_\nu - \bar\tau_2}.$$

To Eqs. (2.4) we must apply additional limitations

$$\int_L \omega_1(\tau)\,d\tau_1 = 0, \tag{2.5}$$

which follow from the uniqueness of the displacements in transversing the L_j ($j = \overline{1, k}$).

From (1.5) and (1.6), with a consideration of Eqs. (1.1) and (2.3), we have

$$D_s r_s(t) + \int_\Gamma k_4(t,\tau) r(\tau)\,ds + \int_L \operatorname{Re}\{k_5(t,\tau)\,\omega_1(\tau)\}\,ds + C_s = f_2(t),\ t \in \Gamma,$$

$$k_4(t,\tau) = -\frac{1}{\pi}\operatorname{Im}\left\{\sum_{\nu=1}^{2} T_\nu(\vartheta)\left[A_\nu(\tau)\ln(\tau_\nu - t_\nu) + l_\nu \overline{A_1(\tau)}\ln(\bar{\tau}_1 - s_\nu t_\nu) + n_\nu \overline{A_2(\tau)}\ln(\bar{\tau}_2 - m_\nu t_\nu)\right]\right\} + k^*(t,\tau),$$

$$k_5(t,\tau) = \frac{1}{\pi i}\left\{T_1(\vartheta)\ln(\tau_1 - t_1)\frac{d\tau_1}{ds} - a(\tau)\,T_2(\vartheta)\ln(\tau_2 - t_2)\frac{d\tau_2}{ds} - \right.$$

$$- \overline{b(\tau)}\,\overline{T_2(\vartheta)}\,\overline{\ln(\tau_2 - t_2)}\frac{d\bar{\tau}_2}{ds} - \frac{d\tau_1}{ds}\sum_{\nu=1}^{2}\bar{l}_\nu \overline{T_\nu(\vartheta)}\,\overline{\ln(\bar{\tau}_1 - s_\nu t_\nu)} +$$

$$\left. + a(\tau)\frac{d\tau_2}{ds}\sum_{\nu=1}^{2}\bar{n}_\nu \overline{T_\nu(\vartheta)}\,\overline{\ln(\bar{\tau}_2 - m_\nu t_\nu)} + \overline{b(\tau)}\frac{d\bar{\tau}_2}{ds}\sum_{\nu=1}^{2} n_\nu T_\nu(\vartheta)\ln(\bar{\tau}_2 - m_\nu t_\nu)\right\},$$

$$f_2(t) = w^0(t, \vartheta_s) - \frac{P_s^1}{E_s F_s}(t + \rho_s), \tag{2.6}$$

$$k^*(t,\tau) = \delta_{ls}\rho_s(\eta - \xi)[\operatorname{sign}(\xi - \eta) + 1]/(2E_sF_s),\ \tau = t^l(\eta),\ t = t^s(\xi)$$

$$(l,\ s = \overline{1, m}),$$

$$D_s = \begin{cases} \Delta_s(G_s d_s)^{-1}, & t \in \Gamma_s', \\ q^s d^2 (d_s)^{-1}, & t \in \Gamma_s'',\ r(t) = r_s(\bar{t}_i^s) = \text{const},\ t \in \Gamma_s'' \end{cases}$$

($\delta_{\ell s}$ is the Kronecker delta).

The unknown constants C_s ($s = \overline{1, m}$) can be determined from the equilibrium conditions for the s-th rib:

$$\int_{\Gamma_s} r_s(\tau)\,ds = P_s^1 - P_s^2\ \ (s = \overline{1, m}). \tag{2.7}$$

The system of integral equations (2.4) and (2.6), along with the auxiliary conditions (2.5) and (2.7), gives a unique solution to the problem. The integrands $k_j(t, \tau)$ ($j = \overline{1, 5}$) can have no more than a weak singularity, in view of the simplifying assumptions. The resultant determining system of equations remains valid, if at some part of the line Γ there is no connection between the plate and the rib via the bond or pins, for example, as a result of failure of the bond or pins due to the nearness of a crack and its passage under the rib. Here a debond zone is formed, whose dimensions can be found, for example, by using the equality of the shear strains in the bond at the ends of the debond section with the maximum allowable strain γ^0, which can be determined experimentally [7]. Here the system (2.4)-(2.7) must be augmented by the condition $r_s(t)/(G_s d_s) = \gamma^0,\ t \in \Gamma$. We will assume that the dimensions of the debond zone are given when the crack passes under a rib.

3. The behavior of the system (2.4)-(2.7) is known near singular points, which correspond to the ends of cracks and the ends of bonded sections [4, 8, 9]. Therefore, the desired functions $\omega_{1j}(t)$ and $r_s(t)$ can be represented in the form

$$\omega_{1j}(t) = \frac{\Omega_j(\beta)}{\sqrt{1-\beta^2}},\quad t = t^j(\beta) \in L_j\ \ (|\beta| < 1,\ j = \overline{1, k}),$$

$$r_s(t) = Q_1^s\frac{(1+\xi)}{2}\left[\ln\left(\frac{1+\xi}{2}\right) + 1\right] + Q_2^s\frac{(1-\xi)}{2}\left[\ln\left(\frac{1-\xi}{2}\right) + 1\right] + R_s(\xi),$$

$$t = t^s(\xi) \in \Gamma_s\ \ (|\xi| \leqslant 1,\ s = \overline{1, m}),$$

where $\Omega_j(\beta)$ and $R_s(\xi)$ are continuous functions in [−1, 1]. The constants Q_1^s and Q_2^s are determined from $R_s(-1)$, $R_s(1)$, and the rigidity parameters of the bonding layer and the rib [8].

An approximate solution of the system (2.4)-(2.7) on sections Γ_s ($s = \overline{1, m}$) can be constructed by a first-order spline fit [10]. To do this, we break up the sections Γ_s ($s = \overline{1, m}$) by the points ξ_j ($j = \overline{1, N}$) and ξ_i^s ($i = \overline{1, N^s}$), such that $t(\xi_j) \in \Gamma_s'$, $t(\xi_i^s) \in \Gamma_s''$, and $t_i^s = t(\xi_i^s)$. The same number of points ξ_j are uniformly distributed in each interval $[-1;\ \xi_1^s - d/(2\rho_s)]$,

$[\xi^s_{N^s} + d/(2\rho_s); 1]$, and $[\xi^s_i + d/(2\rho_s); \xi^s_{i+1} - d/(2\rho_s)]$ $(i = \overline{1, N^s - 1})$ including their end points. The total number of points ξ_j is equal to N. We represent the unknown functions in the form $R_s(\xi) = \sum_{k=1}^{N_\Sigma} R^s_k L^k_s(\xi)$, where $L^k_s(\xi)$ is a function which is continuous in [−1, 1] and linear in each interval $[\xi_{j-1}, \xi_j]$ $(j = \overline{2, N_\Sigma})$, where $N_\Sigma = N + N^s$, and $L^k_s(\xi) = \delta_{kj}$.

We reduce the solution of the problem (2.4)-(2.7) in the usual manner ([4, 8], for example) to a system of linear algebraic equations for the approximate values of the desired functions $\Omega_j(\beta)$ at the nodal points $\beta_i = \cos\left(\frac{2i-1}{N_j}\pi\right)$ $(i = \overline{1, N_j}, j = \overline{1, k})$ and for unknown coefficients R^s_k, $k = \overline{1, N_\Sigma}$, $s = \overline{1, m}$ (we do not present the explicit form of this system because of its complexity).

Having determined $\Omega_j(\beta)$ and $R_s(\xi)$, we find the stresses and displacements in the plate from (1.2) and (2.1), and also the stress-intensity coefficients at the ends of the cracks [4]. The data for isotropic media are obtained by a limiting transformation of the anisotropic parameters $[\mu_\nu \to i, \mu_1 \to \mu_2 \to 0)$ [8].

Below we present calculated results for a plate with cracks and bonded, pinned, or pinned-and-bonded ribs. Figure 2 illustrates the effect of the orientation angle φ of the crack on the stress-intensity coefficients $K_{1,2}(-a)/(\sigma\sqrt{\pi a})$ in an unstrengthened half space and in a half space with two strengthening ribs (curves 1 and 2). The calculations were conducted for boron-reinforced plastic plates with $E_1 = 276.1$ GPa, $E_2 = 27.61$ GPa, $G_{12} = 10.35$ GPa, $\nu_{12} = 0.25$, and h = 2 mm (solid lines); and a glass-reinforced plastic with $E_1 = 53.84$ GPa, $E_2 = 17.95$ GPa, $G_{12} = 8.63$ GPa, $\nu_{12} = 0.25$, h = 2 mm (dashed lines). It was assumed that the principal direction of the orthotropy E_1 coincides with the direction of the crack. A tensile load σ is applied to the plate; the relative rigidity of the rib is $U_s = E_1ha(E_sF_s)^{-1} = 0.04$ and of the bond layer $V_s = G_sd_sa(E_1h\Delta_s)^{-1} = 0.05$; the half-length of the rib is $\rho_s = 10a$; and the half-length of the debond is $\delta_s = 0$.

The effect of the length of the debond on the stress-intensity coefficients is demonstrated in Fig. 3 for cracks in an infinite boron-reinforced plastic plate, strengthened by one pinned-and-bonded rib located a distance 0.5a from the center of the crack. The problem was solved for the following initial parameters: number of pins on the rib $N^s = 18$, $d_s = a/2$, $d = a/6$, attachment step $c_s = a$, h = 1 mm, and $\varphi = \pi/2$. The relative compliance of the pin and the bond $M_s = q_sd^2G_s/\Delta_s$ was taken to be zero (infinitely rigid pin − solid lines) and $M_s = 10$ (compliant pins − dashed lines). For curve 1 the plate is loaded by tensile stresses σ; for curve 2, identical stresses are applied to the plate and the ends of the rib. It is assumed that, for the most highly loaded region near the crack, debonding develops identically in both directions along the rib. When the debonding passes through the pin, the pin fails. This explains why the stress-intensity coefficients for strengthening with infinitely rigid pins show large jumps, while there is no noticeable change for strengthening with compliant pins.

Table 1 compares results obtained from the method presented here, with data from calculations of a pinned joint from the structureless asymptotic theory of point connections [4]. The calculations were done for an infinite isotropic plate (E = 7200 MPa, ν = 0.33, and h = 2 mm), weakened by a linear cut $L = \{|x| < a, y = 0\}$ and strengthened by a pinned rib with parameters $U_s = 20$, $V_s = 500$, $\rho_s = 10a$, $d_s = a/2$, and $d = a/6$. The number of infinitely rigid pins ($M_s = 0$) on the rib is $N^s = 18$, with a step of $c_s = a$. The table shows values of $K_1(a)(\sigma\sqrt{\pi a})$ as a function of the distance x/a between the center of the crack and the axis of the undamaged or damaged strengthening rib. Column 1 gives the results of our investigation and column 2 gives the calculation from the model in [4]. For each pair of $K_1(a)/(\sigma\sqrt{\pi a})$ values, column 3 shows the magnitude of the relative error in percent. As can be seen from the table, the calculated results for both models practically coincide.

The approach presented above for calculating strengthened anisotropic damaged plates can be used to estimate the effect of a strengthening assembly on the residual strength of panels made from layered composite materials. In view of the various forms and types of damage to a layered composite, there can be a large number of failure criteria. We use the strain criterion [11], which is substantiated by experimental data for a wide spectrum of layered composites with various types of layups. The criterion is based on the assumption that the layered composite with fissure-type damage fails at the ends of the fissure, when the magnitude of the strain in the basic load-carrying layers (that is, layers having the

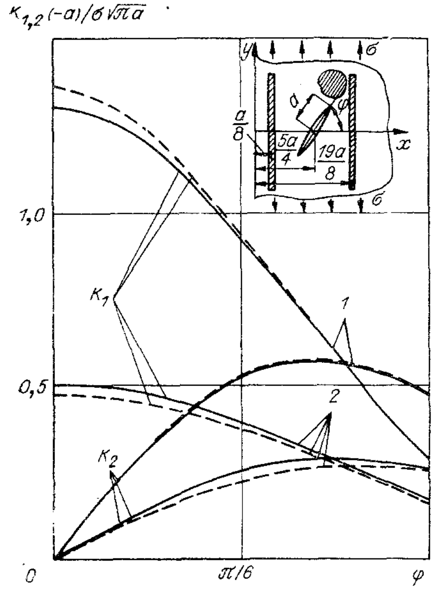

Fig. 2

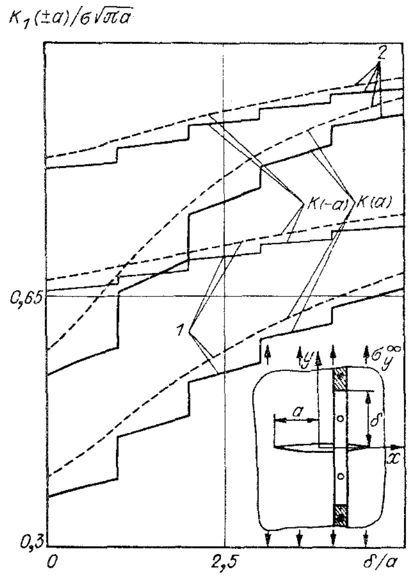

Fig. 3

TABLE 1

x/a	$K_1(a)/(\sigma\sqrt{\pi a})$					
	undamaged rib			damaged rib		
	1	2	3	1	2	3
0	0,8911	0,8858	0,60	1,1145	1,1134	0,10
0,5	0,8699	0,8632	0,78	1,1487	1,1482	0,04
1,0	0,9194	0,9180	0,15	1,1558	1,1577	0,16
1,5	0,9753	0,9763	0,10	1,0970	1,0969	0,01
2,0	0,9918	0,9922	0,04	1,0474	1,0460	0,13

largest angle to the direction of the applied load) reaches its limiting value. Figure 4 shows the critical load σ^*, applied to the plate as a function of the type of layup. The calculations were conducted for an infinite plate of thickness h = 1 mm made from carbon-reinforced plastic with monolayer parameters E_1 = 141 GPa, E_2 = 9.5 GPa, G_{12} = 5.2 GPa, and ν_{12} = 0.31 for various layups $(0/\pm\alpha/90°)_s$, where α is the angle between the x-axis and the fiber direction in the monolayer. The plate has crack-type damage with a half length a = 2, 4, and 6 mm (curves 1-3, respectively) and is strengthened by three bonded ribs (with no debonds) or half-length 20 mm at a distance of 5 mm from each other. The relative rigidity of the ribs and the bond is U_s = 0.4 and V_s = 0.5. The ribs are assumed undamaged (solid curves) and broken in the middle (dashed curves). The generalized rupture viscosity parameter $Q_c = \varepsilon_1\sqrt{2\pi r}$ is taken to be 1.5 $mm^{-1/2}$ according to [11] for all layup types. Analysis of the curves leads to the conclusion that for undamaged ribs the critical load σ^*, is increased somewhat when the end of the crack goes behind the rib. For broken ribs, σ^*, decreases as the crack line grows, and σ_* decreases faster, as α becomes larger.

Figure 5 shows the results of a calculational and experimental investigation of the growth of a fatigue crack from cyclic tension in a 2 mm thick plane specimens made from the aluminum alloy D16AT (σ^{max}_{gross} = 10 kg/mm²) and σ^{min}_{gross} = 1 kg/mm²), both for the plate in its initial state and with strengthening elements: a crack retainer with transverse dimensions 15 × 2 mm from the D16AT alloy and a wide titanium strap 50 × 0.5 mm. The elements were bonded with VK-9. The specimens had stress concentrators (a hole 5 mm in diameter in the center of the specimen with initiated cracks 1-2 mm long). In the calculations with the model described previously, the wide strap was interpreted as several adjacent ribs with equivalent rigidity. In order to determine the crack growth velocity in the bonded specimens, the Paris formula is used with empirical constants taken from tests on the initial specimens. The calculated results showed good agreement with the experimental data. For a crack 100 mm long, the difference between the calculated and the experimental number of load cycles with two retainers did not exceed 2%, but close to 4% with three. The excess of the calculated result over the experimental data can be explained by the physical nonuniformity of

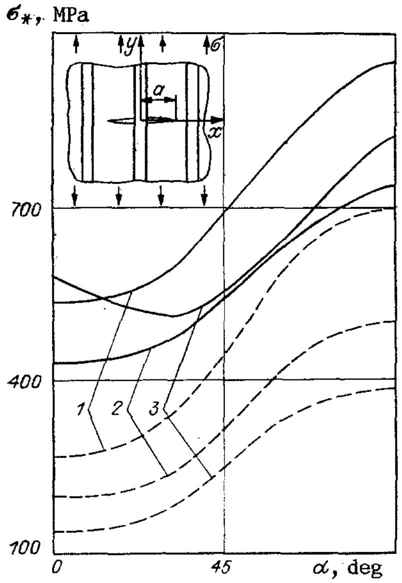

Fig. 4

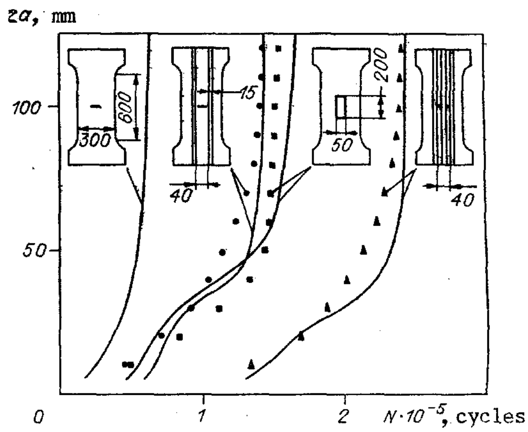

Fig. 5

in the nature of the calculated curve and the experimental points follows from the assumed the bond layer (microdefects), which were not considered in the calculations. The differences calculational model of the contact along the length. We note that on the average the survivability was 2.2 times higher for the specimens with two retainers as compared to the initial specimens, 3.7 times higher for three retainers, and 2.4 times for specimens with the strap.

Analysis of the results presented lead to the conclusion that the developed method for estimating the residual strength and the rupture time is trustworthy and dependable, and also that using bonded and pinned-and-bonded strengthening elements is highly effective for slowing fatigue cracks.

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